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Analysis of the asset replacement level with an uncertain salvage value: a two-factor model

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The aim of this article is to analyse the asset replacement problem from the perspective of optimal replacement level, given a specific tax environment and depreciation policy. Using a real options approach, our model minimizes current operation and maintenance costs and allows the definition of a new value of the replacement flexibility within a multi-cycle environment. The innovation on the valuation process comes from adding an autonomous salvage value’s factor. The results from partial differential equations reveal relevant differences from those observed in one-factor models, especially with regard to optimal replacement levels and in the non-monotonous effects of salvage value variation. The numerical case study also confirms that the salvage value is indeed a worthwhile element in the replacement process. It was possible to determine that, in terms of the magnitude of the cost replacement level, the key roles are played by changes in the speed of mean reversion, as well in the salvage factor volatility. This paper provides some improvements to the existing literature in equivalent annual cost by drawing up a cost minimization problem conditioned by a different salvage value dynamics, and contributes to real options literature by introducing a salvage value factor in the pricing model.

Keywords: real options; replacement theory; salvage value

AMS Subject Classifications: 60H30; 91B25; 91B28; 91B70

1. Introduction

Salvage value can be seen as the projected resale value of any asset, after it has reached the expiry of economic life to a specific owner. Its relevance arises out of the need for asset valuation, as understanding the concept of salvage value involves realizing what is meant by depreciation. The economic value of any asset object depends on the period of its usage, its physical condition and the degree of its second-hand usefulness. There are several real-world applications of markets for second-hand assets (vehicles, buildings, production equipment, aircraft and ships) that show the magnitude of the phenomenon of reusing durable real assets. In these markets, the most interesting aspects to analyse are the salvage value, liquidity and market specificity level of information. There have

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been several papers that explore the asset owner gains from an ex-ante perspective regarding the appropriate transaction level.

Additionally, a fundamental aspect of a replacement decision is assessing the owner benefits in advance, since such information will help define whether the owner will actually decide to replace the asset. Numerous studies have discussed the benefits of correct asset replacement, with capital efficiency having been recognized as one of the issues of replacement efficiency. Shleifer and Vishny’s [1] and Banerji’s [2] work provide examples of empirical studies that analyse the salvage value and its relationship with market progress. Pulvino [3] determined that capital constraints cause firms to liquidate assets at a discount and Chan et al. [4] considered the application of a modified two-cycle replacement model at a railway corporation.

This paper approaches the asset replacement problem, perhaps the most frequent decision made in capital budgeting, using a real options approach, which minimizes the operation and maintenance costs and determines the new cost replacement level within a multi-cycle environment. Calculating salvage value is an ordinary (accounting and market) exercise and a relevant part of the asset replacement problem. There is a relationship between salvage value and the depreciation of asset value and both have importance for tax deductions. The innovation in our valuation process comes from considering an autonomous salvage value’s factor. Traditionally, analysing asset replacement decisions implies an estimation of the net present value/cost associated with an infinite sequence of asset life cycles. One example of such replacement analysis is given by Navon and Maor [5], who analyse the impact of investment timing variation on the net present value.

An alternative approach implies the use of the deterministic Equivalent Uniform Annual Cost (EUAC) of the two assets in competition. This method assumes cost structure consistency, deterministic cash flows and ex-ante knowledge about the salvage value. One of the major problems of the EUAC approach comes from improperly considering the existence of uncertainty. Ierapetritou and Zukui [6] show that ignoring the cost uncertainty produces biased results in the budget process and creates deviations in the execution process. Kierulff [7] recognizes that salvage values have separate risk and inflation implications, which are diverse from their own cash flows and which work in different ways with challengers and defenders. The main issue of uncertainty in an asset replacement problem is that it requires an approach which integrates the new information on current decisions and concurrently assesses the option price of updating the replacement decision timing.[8] Dias and Shackleton [9] show that traditional approaches to replacement investment decisions lead to non-optimal decisions in a stochastic interest rate environment.

The real options approach derives from the similarity between the asset replacement and American call option decisions.[10] The owner of an American call option holds the right to purchase an underlying stock at an exercise price, while the owner of a real asset holds the right to acquire a new asset at a given purchase price. The application of real options to fast technological environments has shown that it is improper to use a single option type in situations involving repeated substitutions.[11]

Rust [12] considers uncertainty by adopting the presumption that a higher cost suggests a larger asset decline and sees modelling cost dynamics as following an arithmetical Brownian motion with constant drift and variance. In the same way, Ye [13] considers a situation where all assets are stochastically comparable, and all have the same initial costs. This author introduces a new concept of physical deterioration that
increases stochastically and returns to its initial state, requiring a replacement each time. Arboleda and Abraham [14] corroborate this by stating that higher deterioration rates can increase the maintenance costs, as a result of the incremental frequency of the preservation routines. Mauer and Ott [15] improved previous replacement models by modelling the operation and maintenance costs using geometric Brownian motion (GBM). This process avoids the deficiencies of the arithmetical Brownian motion, which assumes negative values for costs. One of the ways to improve this model is to consider the exponential estimates as used by Lee and Paxson [16]. Our paper does not follow that line of analysis because of the issues related to the principle of homogeneity.[17] An alternative perspective is given by Myers and Majd [18] and McDonald and Siegel [19] who analysed this problem through the behaviour of the abandonment option facing the project’s residual value or its best alternative. These replacement problems are similar to other real options problems, as the timing option and switch inputs or outputs in the production can be evaluated in a similar way. The main obstacle to such approaches results from the difficulty of determining an asset’s value. Sometimes, these values can be estimated from spot/forward prices, as in the model of Brennan and Schwartz [20], which prices a gold mine with information recovered from the gold markets.

Our paper considers the case, where the operation and maintenance costs decay continuously according to a Wiener process and the salvage value increases stochastically. This model has the convenience of avoiding the need for explicitly defining the relation between the operation and maintenance costs and the salvage value. The challenge is to minimize the expected accumulated discounted cost, considering a variable salvage factor. Thus, after taking a partial differential equation with proper boundary conditions, the model obtains an optimal replacement policy. The contribution of our model is, as Clark and Rousseau [21] put it, to provide an approximation of reality and an accurate way of concretizing the replacement equipment investment, so diminishing the amount of deviation from the budget.

The current section reviews and establishes the line of research based on existing literature. The next section describes the two-factor replacement model in detail. In Section 3, a numerical case study shows the data obtained from the theoretical model. This section also checks the model’s robustness by using sensitivity analysis. Section 4 ends the paper, presenting some conclusions about the outcomes and explaining the research contributions.

2. Replacement model

Generally, classic replacement analysis begins by using a single-asset model that considers salvage value as a function of the operation and maintenance costs. Our model sets a salvage value function to trigger the asset replacement. The salvage value function \( \zeta(C_t, S_t) = C_t S_t \) reflects a market principle that costs, and salvage values are inversely proportional (when operational and maintenance costs increase, the salvage value decreases). Instead of assuming a constant ratio \( \kappa (S_t = \zeta(C_t, S_t)C_t^{-1} = (\kappa)C_t^{-1}) \), our model assumes the existence of useful and realistic mean-reversion process. Fleten and Nasokkala [22] propose a two-factor model to study the investments in gas-fired power plants with reference to stochastic electricity and natural gas prices. They use the mean reversion process to analyse opportunities to abandon the capital equipment. The proposed diffusion process is itself a hybrid of the Black–Scholes stock price model
and Vasicek interest rate model. Therefore, significant changes in \( \zeta(\cdot) \) indicate relevant variations in operation and maintenance costs \( (C_t) \) or in salvage value \( (S_t) \). As a result, the replacement of the asset should be observed when \( \zeta(\cdot) \) exceeds a certain level \( \zeta^*(\cdot) \), either by an increase in \( C_t \) or in \( S_t \). Thus, each time \( \zeta(\cdot) \) reaches a trigger level, it triggers the replacement of the current asset by another stochastic equivalent. For a risk-adjust rate \( r_{an} \), the valuation expression is as follows:

\[
V(C_t, \zeta_t, t) = \min_{\zeta_t} E \left[ \int_0^\infty (C_t(1 - \tau) - \tau \delta^\varphi \vartheta(C_t, t))e^{-r_f \tau} d\tau \right],
\]

where \( E \) is the expectation operator with respect to a probability distribution \( P \) governing \( (C_t, t \geq 0) \) as the Brownian motion started at \( C_N \). The asset valuation results from the difference between the after-tax costs \( C_t(1 - \tau) \) and the tax shield \( \tau \delta^\varphi \vartheta(C_t, t) \). In expression (1), \( C_t \) corresponds to the operation and maintenance costs, \( \tau \) is the tax rate and \( \delta^\varphi \) represents the depreciation rate. By considering \( Z = \alpha_C - (1/2)\sigma_C^2 \), the minimal cost \( C_N \) and adopting an infinite time horizon, it is possible to relax the dependence between \( t \) and the function \( V(\cdot) \), holding a book value \( \vartheta(C_t) = P(1 - \varphi) \left( C_t/C_N \right)^{-\delta^\varphi / 2} \) with \( P \) as the acquisition price and \( \varphi \) as the investment tax credit rate. The role of salvage value factor is given by Equation (6). Like Mauer and Ott [15], we consider the diffusion processes \( dC = \alpha_C C dt + \sigma_C C dW_C \), where \( \alpha_C, \sigma_C > 0 \) are the drift and volatility of the cost \( C \) and \( dW_C \) is an infinitesimal increment in a standard Weiner process, where \( dW_C = \epsilon_t \sqrt{dt} \) and \( \epsilon_t \sim N(0, 1) \). This diffusion process is usually used for modelling stock prices. Thus, we believe that operation and maintenance costs actually behave like stock prices. To estimate the salvage factor, the model uses the mean reversion process \( d\zeta = \mu(\zeta - \bar{\zeta}) dt + \sigma_\zeta \zeta d\zeta \), where \( \mu, \sigma_\zeta > 0 \) are the mean reversion rates and volatility of the salvage factor \( \zeta, \bar{\zeta} \) is the standard level of the salvage factor and \( d\zeta \) is an infinitesimal increment in a standard Weiner process, where \( d\zeta = \epsilon_t \sqrt{dt} \) and \( \epsilon_t \sim N(0, 1) \). This expression originates from the one described by Gibson and Schwartz [23], which represents convenience yield properties as part of oil price evolution, and from another expression proposed by Dixit and Pindyck [10]. There is also strong evidence of a mean reversion presence in the future market and agricultural products as well as some modest evidence of mean reversion in the financial markets.[24]

The Feynman–Kac theorem allows converting a given discounted expectation into a PDE. Contingent claims approach requires that stochastic changes in \( V(\cdot) \) must be spanned by existing assets in the economy. Thus, we assume that it is always possible to find a dynamic portfolio of assets whose price is perfectly correlated with \( V(\cdot) \) [10]. Merton [25] provides an arbitrage-based derivation of the general equation for contingent claims approach and Constantinides [26] gives a derivation based on a continuous time capital asset pricing model. This free-arbitrage assumption permits to obtain the Feynman–Kac stochastic representation formula:

\[
\frac{1}{2} \sigma_\zeta^2 C^2 V_{CC} + \alpha_C C V_C + \rho C \sigma_C \sigma_\zeta C V_{C\zeta} + \frac{1}{2} \sigma_\zeta^2 V_{\zeta\zeta} + (\mu(\zeta - \bar{\zeta})) V_\zeta + C(1 - \tau) - \tau \delta^\varphi P(1 - \varphi) \left( \frac{C}{C_N} \right)^{-\delta^\varphi / 2} = r_f V
\]
It is assumed that in the absence of arbitrage, $V(\cdot)$ solves the partial differential equation (2), which contains the partial derivatives of $V(\cdot)$ with respect to the variables $C$ and $\zeta$, the risk-adjusted drift rate of costs $\alpha_C = r_f + \delta_C$, the mean reversion rate $\mu(\zeta - \zeta)$, the risk free interest rate $r_f$ and convenience yield $\delta_C$. The general solution to Equation (2) is (Appendix 1):

$$V(C, \zeta) = V_H(C, \zeta) + k_A C + k_B C^\zeta, \quad (3)$$

with $V_H(C, \zeta)$ described by:

$$V_H(C, \zeta) = C^{\zeta(1-\beta_1+k_1^\zeta)} \zeta^{\beta_1+k_1^\zeta} [k_D H(h_1, h_2, h_3) + k_EL(l_1, l_2, l_3)], \quad (4)$$

where $k_C, k_D, k_E$ are constants and $k_{A,B}$ is defined as:

$$k_A = \frac{1 - \tau}{r_f - \alpha_C^\zeta}; \quad k_B = - \frac{1}{r_f - \alpha_C^\zeta} + \frac{1}{2} \frac{\zeta \delta^2 P(1 - \varphi)}{1 - \frac{1}{2} \zeta^2 \sigma_C^2} \quad (5)$$

$H(h_1, h_2, h_3)$ is a hypergeometric function (Appendix 2), $L(l_1, l_2, l_3)$ is a Laguerre polynomial (Appendix 3) and $\sigma_C$ corresponds to the standard deviation of $\zeta, C$. In order to reach the general solution of $V(C, \zeta)$, we need to determine constant values using the following boundary equations. The first requirement, commonly called the value matching condition, ensures the equality condition between the options to abandon and invest at the critical level. This condition also ensures the function’s continuity by neglecting the difference in making an investment immediately before or just after the value passes the critical level:

$$V(C^*, \zeta^*) = V(C_N, \zeta_N) + P(1 - \varphi) - \frac{\zeta^*}{C^*} - \tau \left\{ \frac{\zeta^*}{C^*} - \theta(C^*) \right\}. \quad (6)$$

Apart from ensuring value-matching at critical level $(C^*, \zeta^*)$, we need to define an equal slope between the pay-off and the function $V(\cdot)$:

$$V_C(C^*, \zeta^*) = V(C_N, \zeta_N) - \tau \theta_C(C^*); \quad V_\zeta(C^*, \zeta^*) = V_\zeta(C_N, \zeta_N) - 1 + \tau,$$

with the derivative in the order to $C$ of book value $\theta_C(C)$:

$$\theta_C(C) = \frac{\delta^a}{P(1 - \varphi)} \left( \frac{C}{C_N} \right)^{-\frac{C^*}{C_N}} \quad (7)$$

When costs reach very high values, it is likely that an asset could be replaced before it is completely written off. To discover the value of such growth, we will consider an expression representing the present value of the expected costs:

$$V(C, \zeta) = (1 - \tau) \int_0^\infty e^{-r_f t} C e^{\xi t} - \tau \delta^a P(1 - \varphi) \left( \frac{1}{C_N} \right)^{\xi} \times \int_0^\infty e^{\xi t} C \exp \left\{ \left( \frac{\xi x_C^*}{2} + \frac{1}{2} \xi (\xi - 1) \sigma_C^2 t \right) \right\} \quad (8)$$
\[ V(C, \xi) = \frac{(1-\tau)}{\delta_C} C - \frac{\tau \delta^a P(1-\varphi)(1/\xi)}{r_f - \xi \sigma_C^2 - \frac{1}{2} \xi(\xi-1)\sigma_C^2} C^{\xi} \]  

being \( \xi = \delta^a / Z \) and \( \delta_C \) representing the convenience yield. Knowing that \( \xi < 0 \) and calculating the partial derivative \( V_C \):

\[
\lim_{C \to \infty} \left( \frac{(1/\xi)}{r_f - \xi \sigma_C^2 + \frac{1}{2} \xi \sigma_C^2 - \frac{1}{2} \xi^2 \sigma_C^2} C^{\xi} \right) = 0
\]

that results in the boundary equation \( \lim_{C \to \infty} V_C(C, \xi) = \frac{1-\tau}{\delta_C} \). When salvage value \( S \) reaches zero and salvage factor \( \xi \) takes equal value, the accumulated cost function is no longer affected by its salvage value. Thus, \( V(C, 0) = V(C) \).

3. Solutions and robustness analysis

The model described above involves a substantial number of parameters for calculating the cost replacement level. Numerical models are often criticized because of their particular assumptions regarding parameter values. We tried to use some empirical support for specifying the parameter values. The model was solved under different assumptions in order to assess its robustness. Thus, the sensitivity analysis was conducted by using a systematic numerical analysis of a case study based on the B737 aircraft replacement process. Apart from assessing the model’s robustness, the main objective was to quantify the impacts on the cost replacement level of changing some parameter values (Table 1).

Recalling the model design, we assumed a two-factor cost function, where one of factors (salvage factor) follows a mean-reverting process and the other one follows the path of GBM. From the market analysis,[27] we have built Figure 1 that represents the evolution of cost index and salvage value index during 25 years. Figure 1 also contains the behaviour of the salvage factor index.

For the initial standard salvage factor \( \xi_N \) was adopted a value that represents 86% of the asset’s purchase value and corresponds to its average value during the first year of activity. We also considered the standard deviation of the salvage factor \( \sigma_\xi \) as being higher than the standard deviation of the cost \( \sigma_C \). The value proposed that the mean

<table>
<thead>
<tr>
<th>Cost parameters</th>
<th>Salvage factor parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_C )</td>
<td>Drift rate</td>
</tr>
<tr>
<td>( \sigma_C )</td>
<td>Volatility</td>
</tr>
<tr>
<td>( C_N )</td>
<td>Initial</td>
</tr>
<tr>
<td>( P )</td>
<td>Asset purchasing price</td>
</tr>
<tr>
<td>( \delta^a )</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( r_f )</td>
<td>Risk free interest rate</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Market risk price</td>
</tr>
</tbody>
</table>

Table 1. Parameters used in case study.
The reversion rate $\mu$ is in accordance with the competitive characteristics of the aircraft market. In order to be able to compare the results obtained in this model and quantify the relevance of the differences, the case study related to the models described by Mauer and Ott [15] (model I) and Zambujal-Oliveira and Duque [28] (model II) was replicated. This procedure is methodologically defensible because both models are structurally similar and differ only with regard to the process the salvage value is modelled on. In order to understand the model better, we present below the simulation of the cost function with two dimensions for model I and model II (Figure 2).

Given the assumption of a constant tax regime and a two-factor cost function, the model obtained a cost replacement level of $V^* = US 418,436$. This replacement level reveals significant changes, as it denotes a deviation of 15.37% from the standard value.

Moreover, model III shows some robustness, since the values of the operation and maintenance costs ($C^* = US 3378$) remain very similar to the standard values ($\Delta C^* = 1.45\%$), but the salvage value ($S^* = US 30,874$) shows variations of the same order of magnitude of $V^*$ ($\Delta S^* = -14.34\%$).

From Table 2, it is possible to see a relevant difference between one-factor and two-factor models. Model I produces an optimal level with high costs and low salvage values and models II and III show evidence of lower optimal levels of cost and higher salvage values. The accumulated cost values of models II and III have completely different magnitudes relatively to model I, in result of capturing the uncertainty related with the salvage value. As model III produces higher accumulated cost values, it seems to demonstrate, for this numeric case, that the reversion mean process cannot be used with vantage. These results also appoint to an early replacement of the asset when using models II or III.
Following this, we performed a sensitivity analysis on the most relevant parameters for the replacement level by use of perturbation for those parameters, while all others stayed identical, as in the base valuation of the replacement level. Thus, we oscillated the values of mean reverting rate by 10%, with $l$ as standard value, and a standard deviation value of $\sigma$. The sensitivity analysis results, by base valuation, are reported in Table 3, where it is revealed that the three parameters have an important effect on the replacement level.

Figure 2. Simulation of the processes contained in model II and model III.

Table 2. Solutions from the different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>Var% (I,III)</th>
<th>Var% (II,III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^*$</td>
<td>$8787 M$</td>
<td>$3330 M$</td>
<td>$3378 M$</td>
<td>$-61.6$</td>
<td>$-1.5$</td>
</tr>
<tr>
<td>$S^*$</td>
<td>$9415 M$</td>
<td>$36,042 M$</td>
<td>$30,874 M$</td>
<td>$227.9$</td>
<td>$-14.3$</td>
</tr>
<tr>
<td>$V^*$</td>
<td>$83,046 M$</td>
<td>$369,141 M$</td>
<td>$425,878 M$</td>
<td>$412.8$</td>
<td>$15.4$</td>
</tr>
</tbody>
</table>

I: Mauer and Ott [15]; II: Zambujal-Oliveira and Duque [28] and III: Current model.

Following this, we performed a sensitivity analysis on the most relevant parameters for the replacement level by use of perturbation for those parameters, while all others stayed identical, as in the base valuation of the replacement level. Thus, we oscillated the values of mean reverting rate by 10%, with $l$ as standard value $\zeta_S$ and a standard deviation value of $\sigma$. The sensitivity analysis results, by base valuation, are reported in Table 3, where it is revealed that the three parameters have an important effect on the replacement level.

Table 3 (Panel A) contains the changes in the accumulated cost $V^*$ from changing the mean reverting rate $\mu$. Given the lack of the variance of the factor of long-term technology, Paxson [29] stated that the incentive for increasing investment is related to the increase in the speed of mean reversion. Accordingly, when $\mu$ takes higher values,
there is a faster return to the average value of the salvage factor, increasing the cost replacement level (3.80%). This higher value of the cost replacement level outcomes indicates a decrease of nearly 9% in the salvage value $S^*$. An oscillation of 10% in the speed of the mean reversion causes an effect in the cost replacement level that corresponds to the doubling of the effect in the cost value $C^*$.

Given a standard salvage factor $\zeta_S$, we tested the model’s response to a 10% variation in this parameter. As established in the literature, when $\zeta_S$ takes higher values, a gap enlargement of between $C$ and $S$ values will be expected because of their complementary. For similar reasons, lower values of $\zeta_S$ will tend to limit the range between $C$ and $S$. Table 3 (Panel B) contains the impact on the critical level of the variables that integrate the replacement process resulting from changes in the standard factor $\zeta_S$. Its observation allows us to collect evidence that the lower values of cost $C^*$ and the thinner replacement periods result from higher values of $\zeta_S$. The reason associated with this behaviour is that higher levels of salvage value joined with lower costs make investing in assets more attractive. We were expecting a similar effect from introducing volatility $\sigma_S$ but in this case, variation will depend on the magnitude of $\sigma_C$ and $\sigma_S$ relationship. This yields the assumption that $\sigma_S > \sigma_C$ controls the direction of critical cost variation $C^*$. As Dobbs [30] states, a periodic asset exchange accelerates from including $r_1$ and reducing the replacement critical level.

Table 3 (Panel C) shows the relevant effects on the cost replacement level $V^*$ of changes in the salvage factor’s volatility. The effect on the $V^*$ is bigger in terms of down variations (+10.8%) than up variations (−5.68%). As a result, it is possible to note that lower critical values $C^*$ (−2.36%) result from higher volatility $\sigma_S$ values. A possible explanation for this behaviour seems to be that higher volatility levels of salvage factors could create more opportunities for reaching earlier optimal replacement levels, either by increasing cost value or by increasing salvage value.

By comparing the three panels, it is possible to find out different signal effects from the perturbations in all parameters. A negative oscillation in $\mu$ produces negative effects in the cost replacement level $V^*$. In the other two panels, the same oscillation

<table>
<thead>
<tr>
<th>Panel A</th>
<th>$\Delta \mu$</th>
<th>$C^*$ (SM)</th>
<th>$\Delta C^*$ (%)</th>
<th>$S^*$ (SM)</th>
<th>$\Delta S^*$ (%)</th>
<th>$V^*$ (SM)</th>
<th>$\Delta V^*$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>−10p.p.</td>
<td>3346</td>
<td>−0.95</td>
<td>33,952</td>
<td>9.97</td>
<td>418,436</td>
<td>−1.75</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>3378</td>
<td>0.00</td>
<td>30,874</td>
<td>0.00</td>
<td>425,878</td>
<td>0.00</td>
</tr>
<tr>
<td>0.6</td>
<td>+10p.p.</td>
<td>3441</td>
<td>1.87</td>
<td>28,216</td>
<td>−8.61</td>
<td>442,076</td>
<td>3.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>$\Delta \zeta_S$</th>
<th>$C^*$</th>
<th>$\Delta C^*$ (%)</th>
<th>$S^*$</th>
<th>$\Delta S^*$ (%)</th>
<th>$V^*$</th>
<th>$\Delta V^*$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2</td>
<td>−10%</td>
<td>3383</td>
<td>0.16</td>
<td>30,347</td>
<td>−1.71</td>
<td>426,925</td>
<td>0.25</td>
</tr>
<tr>
<td>8</td>
<td>0%</td>
<td>3378</td>
<td>0.00</td>
<td>30,874</td>
<td>0.00</td>
<td>425,878</td>
<td>0.00</td>
</tr>
<tr>
<td>8.8</td>
<td>+10%</td>
<td>3374</td>
<td>−0.11</td>
<td>31,235</td>
<td>1.17</td>
<td>425,142</td>
<td>−0.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C</th>
<th>$\Delta \sigma_S$</th>
<th>$C^*$</th>
<th>$\Delta C^*$ (%)</th>
<th>$S^*$</th>
<th>$\Delta S^*$ (%)</th>
<th>$V^*$</th>
<th>$\Delta V^*$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>−10p.p.</td>
<td>3400</td>
<td>0.66</td>
<td>29,363</td>
<td>−4.89</td>
<td>471,775</td>
<td>10.78</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>3378</td>
<td>0.00</td>
<td>30,874</td>
<td>0.00</td>
<td>425,878</td>
<td>0.00</td>
</tr>
<tr>
<td>0.3</td>
<td>+10p.p.</td>
<td>3298</td>
<td>−2.36</td>
<td>37,242</td>
<td>20.63</td>
<td>401,704</td>
<td>−5.68</td>
</tr>
</tbody>
</table>

Notes: $\mu$: mean-reversion rate; $\zeta_S$: standard level of salvage factor; $\sigma_S$: standard deviation of salvage factor; $C^*$: critical level of costs; $S^*$: critical level of salvage value and $V^*$: project value at critical level.
produces positive effects. The magnitude of the effects for the same perturbation is maximum (10.78%) in the negative oscillation of the volatility of the salvage factor $\sigma_S$ and minimum ($-0.17\%$) in the positive oscillation of the standard salvage factor $\tilde{\sigma}_S$.

4. Conclusions

The operational asset replacement is usually driven by the deterioration of the asset itself, leading to cost decreases through efficiency boosts. To assure a continuous production process, determining the replacement level of each operational asset is crucial. Defining the cumulative cost level can be problematic as randomness may lead to oscillations in operations as suppositions of constant operation may lead to inappropriate and overpriced replacement policies.

Our work demonstrates a new asset replacement policy based on the ability to measure the salvage value’s hidden flexibility. It enhances the previous literature that mainly determines the replacement level using solely a one-factor function. In that way, this document strengthens the cost function theory, introducing an innovative dynamic salvage factor. It redefines the constant factor (the constant that balances the relation between cost and salvage value) as a mean reversion process. This way of addressing the problem of asset replacement assumes a multi-cycle environment and a stationary tax regime and relaxes the assumption of first-degree homogeneity.

The case study permits assessing the model’s robustness and quantifies the impact of parameters’ variation. Considering that the fact of the operation and maintenance costs had remained almost steady (the highest variation was around 1%) despite the fluctuations in parameters, we can consider the model robust. The deviations in the cost replacement level attest to the relevance of the introduction of the dynamic salvage factor. The evidence collected from the case study indicates that the model detects the volatility associated with the second-hand market as a very significant aspect in the definition of the replacement level. The speed of market adjustment to a given price does not significantly influence the replacement level. The same consideration applies to the standard level of the salvage factor.

The tax consideration influences the replacement decisions. There is a particular interest in studying the relative sensitivity of the model to operating costs, resale values and comparative tax levels.[31] Our next step will be to extend our study to an uncertain tax regime environment.

References


Appendix 1. General solution

To determine the general solution of the homogeneous equation (2), our paper used the method of characteristics [32] to accomplish a canonical form with a new coordinate system. This step made possible the definition of a partial differential equation with separated variables.[33] Firstly, we considered the general form of the second-order partial differential equation
\[ aV_{CC} + 2bV_{C} + cV_{zz} + dV_{z} + eV_{z} + fV = g \]
with the equation coefficients \(a, b, c, \ldots, f, g\). Including the new coordinate system \((\theta, \eta)\), we have settled a function \(V(\theta, \eta)\) whose canonical form is \(V_{\theta\theta} = \phi(\theta, \eta, V, V_\theta, V_\eta)\). From Equation (5), we took the coefficients \(a = \frac{1}{2} \sigma_c^2 C^2\), \(b = \frac{1}{2} \sigma_c \sigma_C \rho C\), and \(c = \frac{1}{4} \sigma_L^2\) and calculated the determinant expression \(b^2 - ac = \frac{1}{4} \sigma_c^2 \sigma_C^2 C^2 (\rho_c^2 - 1)\). Assuming Equation (2) as a parabolic equation, we swapped between coordinates \((C, \zeta)\) to \((\theta, \eta)\) and solved the characteristic equation \(\eta_C + \frac{\sigma_c^2}{\sigma_C} \eta_z = 0\), with \(\eta_C = \frac{\partial \eta}{\partial C}\) and \(\eta_z = \frac{\partial \eta}{\partial z}\). After obtaining \(\frac{dz}{dC} = \frac{b}{a} = \frac{\sigma_c^2 \zeta}{\sigma_C C}\) with \(\frac{dz}{\zeta} = \frac{\sigma_c dC}{\sigma_C C}\) and \(\ln(\zeta) = \frac{\sigma_c^2 \ln(C) + \zeta_0}{C}\), we applied the exponential function to both sides and determined the value of \(\zeta = C^{\frac{\sigma_c^2}{\sigma_C}} e^{\eta}\). As \(\zeta_1 = e^{\eta}\), then \(\zeta_1 = \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C}}}\). Matching \(\eta(C, \zeta) = \zeta_1\) and \(\eta(C, \zeta) = \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C}}}\). Equally, we needed a function \(\theta\) that intercepts other characteristic curve, and thus we considered \(\theta(C, \zeta) = C\). The partial derivatives of \(\eta(\cdot)\) and \(\theta(\cdot)\) with respect to \(C\) and \(\zeta\) can be defined as \(\eta_C(C, \zeta) = -\frac{\sigma_c^2 \zeta}{\sigma_C C}, \eta_z(C, \zeta) = \frac{1}{C^{\frac{\sigma_c^2}{\sigma_C}}}, \theta_C(C, \zeta) = 1, \theta_z(C, \zeta) = 0\), with \(\theta_C = \frac{\partial \theta}{\partial C}\) and \(\theta_z = \frac{\partial \theta}{\partial z}\). Given \(\psi(\theta, \eta) = V(C, \zeta)\), then \(V_C = \theta \psi_0 + \eta \psi_\eta\) and \(V_z = \theta \psi_0 + \eta \psi_\eta\), from which we hold the expressions:
\[
\begin{align*}
V_C &= v_0 - \frac{\sigma_c^2 \zeta}{\sigma_C} - \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C} + 1}} \psi_\eta; \quad V_\zeta = \frac{1}{C^{\frac{\sigma_c^2}{\sigma_C}}} \psi_\eta; \\
V_{CC} &= v_{\theta\theta} - \frac{2 \sigma_c^2 \zeta}{\sigma_C} - \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C} + 1}} \psi_{\theta\eta} + \left(\frac{\sigma_c^2 \zeta}{\sigma_C} - \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C} + 1}}\right)^2 \psi_{\eta\eta}; \\
V_{\zeta\zeta} &= \left(\frac{1}{C^{\frac{\sigma_c^2}{\sigma_C}}}\right)^2 \psi_{\eta\eta}, \quad V_{C\zeta} = \frac{1}{C^{\frac{\sigma_c^2}{\sigma_C}}} \psi_{\theta\eta} - \frac{\sigma_c^2 \zeta}{\sigma_C} \frac{\zeta}{C^{\frac{\sigma_c^2}{\sigma_C} + 1}} \psi_{\eta\eta}.
\end{align*}
\]
Replacing the previous expressions in the equation:
\[
\frac{1}{2} \left( V_C \sigma_C C + V_\zeta \sigma_\zeta \right)^2 = \frac{1}{2} V_{CC} \sigma_C^2 C^2 + V_{C\zeta} \sigma_C \sigma_\zeta C + \frac{1}{2} V_{\zeta\zeta} \sigma_\zeta^2,
\]
the following canonical form was obtained:
\[
\frac{1}{2} (\sigma_c^2 \theta^2 v_{\theta\theta} + \sigma_\zeta^2 \theta v_\theta + \left(\frac{\zeta}{\sigma_C \eta} - \left(\frac{\varphi_C}{\sigma_C \eta}\right)\right) v - \frac{\sigma_c^2 \zeta}{\sigma_C} \eta v_\eta = r_f v
\]
In order to solve the last Equation (12), we used the variable separation method.[33] We introduced some new variables with the following values: \( m = \bar{\theta} \zeta \eta \) and \( n = \eta \). Setting the equivalence \( v(\theta, \eta) = v(m, n) \) yields the expressions:

\[
v_\theta = \frac{\sigma_c}{\sigma} v_m, \quad v_\eta = \frac{\sigma_c}{\sigma} v_n, \quad v_m = \frac{1}{n} v_m + v_n.
\]

After substituting them in Equation (12), it is possible to settle the expression:

\[
\frac{1}{2} \left( \sigma_c^2 \nu_{mm} \right) = - (\zeta - m) \mu \nu_m \left( (\zeta - m) \mu - \frac{\sigma_c^2}{\sigma_c} \frac{\sigma_c}{\sigma} \right) n \nu_m + r_f v. \tag{13}
\]

After carrying out variable switching in order to obtain \( v(m, n) = v(q, r) \), function \( v(q, r) \) takes the following form: \( v(q, r) = Q(q) R(r) \), which represents the product of two functions and permits a solution based on two ordinary differential equations. Adopting equal notation: \( Q' = dq/dq \) and \( R' = dR/dr \) means that we can apply the following equalities:

\[
v_m = v_q = Q'R', \quad v_m = v_q = Q'R'.
\]

Applying these expressions to Equation (13) allows the assembly of an expression like this one:

\[
\frac{1}{2} \left( \sigma_c^2 Q'' + ((\zeta - q) \mu) Q' + \left( (\zeta - q) \mu - \frac{\sigma_c^2}{\sigma_c} \frac{\sigma_c}{\sigma} \right) r Q'R' - r_f Q R = 0, \tag{14}
\]

with a free-risk interest rate of \( r_f \). Manipulating this equation in order to split components that are functions of \( Q \) or of \( R \), we get:

\[
k^2 = \frac{R'}{R} = - \frac{1}{2} \left( \sigma_c^2 Q'' + ((\zeta - q) \mu) Q' - r_f Q \right) \left( \frac{1}{(\zeta - q) \mu - \frac{\sigma_c^2}{\sigma_c}} \right) Q,
\]

with a constant ratio \( k^2 \).[34] This expression allows us to define the following ordinary differential equations:

\[
\frac{1}{2} \left( \sigma_c^2 Q'' + ((\zeta - q) \mu) Q' + \left[ \left( (\zeta - q) \mu - \frac{\sigma_c^2}{\sigma_c} \frac{\sigma_c}{\sigma} \right) \frac{r Q'R' - r_f Q R}{Q} \right] Q = 0; \tag{16}
\]

\[
r Q'R' - k^2 R = 0 \tag{17}
\]

Equation (16) has a general solution of the form [35] \( Q(q) = q^{\beta_1} \left[ K_1 H(h_1, h_2, h_3) + K_2 L(l_1, l_2, l_3) \right] \), where \( K_1 \) and \( K_2 \) are constants. \( H(\cdot) \) is a confluent hypergeometric function pleased with the following parameters:

\[
\beta_1 = - \frac{2 \zeta \mu - \sigma_c^2 - \sqrt{4 \zeta^2 \mu^2 \sigma_c^2 - 4(1 + 2k) \zeta \mu \sigma_c \sigma_c^2 + \sigma_c^2(8r_f \sigma_c + 8k \sigma_c^2 \sigma_c \sigma_c^2)}}{2 \sigma_c^2 \sigma_c^2} \]

\[
h_1 = - \frac{1}{2 \sigma_c^2} \left( 2 \zeta \mu - \sigma_c^2 - 2k \sigma_c^2 \right)
\]

\[
- \frac{1}{\sigma_c^2 \sigma_c^2} \left( 4(\zeta \mu)^2 \sigma_c^2 + 8r_f \sigma_c \sigma_c^2 - 4 \zeta \mu \sigma_c \sigma_c^2 - 8k \zeta \mu \sigma_c \sigma_c^2 + 8k \sigma_c^2 \sigma_c^2 + \sigma_c^2 \sigma_c^2 \right)
\]

\[
h_2 = - \frac{1}{\sqrt{\sigma_c^2 \sigma_c^2}} \left( \left( 4(\zeta \mu)^2 \sigma_c^2 + 8r_f \sigma_c \sigma_c^2 - 4 \zeta \mu \sigma_c \sigma_c^2 - 8k \zeta \mu \sigma_c \sigma_c^2 + 8k \sigma_c^2 \sigma_c^2 + \sigma_c^2 \sigma_c^2 \right) \right)
\]

\[
h_3 = 2q \mu \frac{1}{\sigma_c^2}.
\]
$L(\cdot)$ represents a Laguerre generalized polynomial with these parameters:

$$l_1 = -\frac{1}{2\sigma_z^2} \left( 2\tilde{\gamma} \mu - \sigma_z^2 - 2k\sigma_z^2 \right)$$

$$- \sigma_z^2 \sqrt{\frac{1}{\sigma_C \sigma_z^4} \left( 4\sigma_C (\tilde{\gamma} \mu)^2 + 8\gamma_c \sigma_C \sigma_z^2 - 4\tilde{\gamma} \mu \sigma_C \sigma_z^2 (1 + 2k) + 8k \gamma_c^2 \sigma_z^3 + \sigma_C \sigma_z^4 \right)}$$

$$l_2 = -\frac{1}{\sqrt{\sigma_C \sigma_z^2}} \left( \sqrt{\left( 4\tilde{\gamma}^2 \mu^2 \sigma_C + 8\gamma_c \sigma_C \sigma_z^2 - 4k \mu \sigma_C \sigma_z^2 - 8k \tilde{\gamma} \mu \sigma_C \sigma_z^2 + 8k \gamma_c \sigma_z^3 + \sigma_C \sigma_z^4 \right)} \right) :$$

$$l_3 = h_3 = \frac{2q\mu}{\sigma_z^2}.$$ 

Expanding Equation (17) to seek the function $R(\cdot)$: $\frac{dR}{R} = k^2 \frac{dr}{r}$, $\ln(R) = \ln(r^k) + r_0$, with the constant $r_0$. Applying the exponential function to both sides, we obtain $R(r) = K_r r^k$ for $K_r = e^{r_0}$ and consequently:

$$v(q, r) = q^{\beta_1} r^k [K_d H(h_1, h_2, h_3) + K_b L(l_1, l_2, l_3)] \quad (18)$$

Unfolding the above expression for the original coordinate system:

$$V_H(C, \zeta) = C^{e^{\zeta(1-\beta_1+k^2)} \zeta^{\beta_1+k^2}} [K_d H(h_1, h_2, h_3) + K_b L(l_1, l_2, l_3)] \quad (19)$$

which represents the homogeneous part of the two-factor cost function $V(C, \zeta)$.

**Appendix 2. Hypergeometric function**

$H(\cdot)$ is a hypergeometric confluent function, also known as Kummer’s function. Its value can be taken from the following series expansion $1H_1(a, b, z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(b)_n n!}$, where $a$ and $b$ are integers and $z$ is the variable. The integral form of the function is:

$$1H_1(a, b, z) = \frac{\Gamma(b)}{\Gamma(b-a)\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt \quad (20)$$

with $\Gamma(a) \int_0^\infty t^{a-1} e^{-t} dt$, where $a$ and $b$ represent polynomial degrees.

**Appendix 3. Laguerre function**

The generalized $n$ degree Laguerre polynomial, $L_{n,a}(\cdot)$ is given by:

$$L_{n,a}(z) = \frac{(a+1)}{n!} 1H_1(-n, a+1, \zeta), \quad (21)$$

for $a > -1$. Replacing $t$ for $n$, the Laguerre function becomes:

$$L_{n,a}(z) = \frac{1}{\Gamma(t+1)} 1H_1(-t, a+1, z), \quad (22)$$

whose value corresponds to a multiple of an $n$-degree Laguerre polynomial.