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The voter who wasn't there: Referenda, Representation and Abstention*

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Abstract

We analyze single binary-choice voting rules and identify the presence of the No-Show paradox in this simple setting, as a consequence of specific turnout or quorum conditions that are included in actual rules. Since these conditions are meant to ensure a representative outcome, we try to formalize this concern and reach our main result: no voting rule can ensure representation if abstention is possible, unless specific restrictive assumptions are made on the preference domain of abstainers. We then focus on the main referendum systems and show that appropriate restrictions do make them compatible with representation.

The main purpose of our paper is, however, to provide a tool for referendum design. Unawareness of the fact that defining a representative voting system necessarily entails restrictions on the preference domain of non-voters may lead to the imposition of restrictions that do not satisfy any deliberate criteria and generate such problems as the No-Show paradox. We therefore recommend instead that a conscious choice be made on how abstention is to be interpreted and that this choice be used as a tool to derive the corresponding referendum rule.

JEL Classification: D71

*The idea for this paper started with some jocose but insightful notes written by José João Marques da Silva at the time of the first referendum held in Portugal (1998). Without being aware of the "no-show paradox", he had the right intuition on the pathologies of the Portuguese referendum. José João passed away in August 2000. ISEG lost a bright, interested, clever and a friendly scholar. May we dedicate this paper to his memory. A preliminary version of the paper, titled 'Referenda, Quorum and the Abstention Paradox' was presented at the 2002 Annual Meeting of the Public Choice Society and Economic Science Association, Pasadena, CA and at the 2001 Annual Meeting of the European Public Choice Society, Paris. We would like to thank Moshé Machover, Vincent Merlin, Frank Steffen and Mathew Braham for useful comments. The usual proviso applies.

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1 Introduction

Direct democracy is increasingly popular in European countries¹. Although Switzerland (Frey, 1994) has been using direct democracy procedures for decades, and for a great variety of issues, we now see countries like Italy, Spain and Portugal introducing the referendum institution to decide on matters as relevant as abortion, while countries like Finland and Denmark are also using referenda to decide upon crucial issues dealing with European Integration such as the ratification of treaties, the decision to join the Union, or the adoption of the Euro (Warre 1988, Arter 1995).

Although the use of referenda is widespread, it is far from homogeneous: differences include the right of initiative (group of citizens, Parliament, President or monarch), the object of the question (constitutional amendment, ordinary Act approved by the Parliament, bill proposal, local issue), the intended effects of the referendum (approval or veto), the domain of voters (all electors or electors of particular states) and the rules that ultimately decide the outcome. For the purposes of our paper, we restrict the analysis to the typical referendum, that is held to decide on a unique change to the status quo - and the citizens are called to either ratify or veto that change. From here on, we will simply examine the case where a vote for 'No' is a vote for the status quo whereas a vote for 'Yes' supports the change - and we also study the consequences of this assumption and the extent to which it is costless.

The public and social choice literature has seldom focused on simple Yes-No voting for an equally simple reason: in the presence of just two alternatives - a change versus the status quo -, the Condorcet-Arrow problems of collective choice disequilibrium vanish and most decision-making rules lead to the same outcome. A simple majority rule (associated with some tie-breaking rule) should then be used, ensuring the choice of the Condorcet winner². Nurmi (1997), in his analysis of referenda, concludes that every referendum should indeed be restricted to a unique binary decision but also that its result should be binding, regardless of turnout. In turn, Felsenthal and Machover (1997), defining ternary voting games as a generalization of simple voting games where abstention is an additional option for voters, conclude that the simple majority rule achieves the highest 'democratic participation index' defined as responsiveness to the desires of the average voter, inherently recommending its use in the presence of abstention.

In an apparent paradox, however, several countries still choose to re-

¹Apart from India, Israel, Japan, the Netherlands and the United States (on a federal level), all major democracies have used national referenda.

²The Condorcet winner is the alternative that defeats all others in a pairwise comparison.

sort to referenda where that simple majority requirement is coupled with an additional turnout condition - and only if that condition is met does the majority rule apply. Similarly, a quorum requirement is a widespread feature of decision-making in all types of committees, ranging from legislative bodies and general assemblies to simple department meetings. This type of additional requirement is usually said to stem from the concern with representation - the need to ensure that the actual voters are 'good' representatives of the potential voters - and need not be, therefore, a paradox in itself. However, the inclusion of a turnout condition in addition to the majority condition may create a virtual dilemma for supporters of the status quo: if they do not vote, they may contribute for the approval of the issue, because their abstention may lead 'Yes' supporters to fulfill the majority condition; if they vote (or attend the meeting) they may cause the approval of the issue by contributing to fulfill the turnout condition. The latter possibility constitutes a real paradox in the heretofore paradox-free world of two alternatives: voters may be able to manipulate the voting outcome by abstaining.

This possibility of manipulating outcomes through abstention is the definition of the abstention or 'no-show' paradox, a term coined by Fishburn and Brams (1983) for a setting where three or more proposals are to be voted upon. Lepelley and Merlin (2001) distinguish two relevant cases in that setting: the positive abstention paradox (PAP) where "some voters with a loser a_j ranked first are deleted (or they abstain) and a_j becomes a winner" and the negative abstention paradox (NAP) where "some voters with a winner a_j ranked last are deleted (or they abstain) and a_j becomes a loser" (Lepelley and Merlin 2001 p.55). These paradoxes, however, hinge on the existence of a third alternative. In our case, given that there is only a binary choice to be made, the distinction between PAP and NAP vanishes and the paradox is caused by the addition of the turnout condition to the decision rule. Despite this distinction, the essence of the issue is the same and we will therefore apply the same phrase to describe it.

In this paper, we first try to formalize the concern with a representative outcome and we argue that this concern can only be reconciled with a turnout condition in referenda (or a quorum in committees), or indeed with *any referendum system*, through specific interpretations of abstention. Awareness of this connection might be of relevance in referendum design - rather than defining a rule that is meant to ensure a representative outcome, which results in a certain interpretation of abstention, we suggest a reversal of this process: making a deliberate choice on the appropriate interpretation of abstention (namely, avoiding the aforementioned paradox) and deriving the voting rule from that interpretation. We base our analysis on referenda, although our results extend to decision-making in committees as well as to

other institutional settings involving a simple binary choice.

In section 2, we introduce the formal framework, present a typology of referenda and identify the type that causes the abstention paradox. In section 3, we formalize the notion of representation and develop an axiomatic study of the model, presenting our main impossibility result: in the absence of a specific interpretation of the preferences of abstainers, no rule that is solely based on the preferences of the set of actual voters can simultaneously satisfy representation and consistency with the application of the Condorcet principle³ to the population. Designing a referendum that satisfies these axioms therefore requires some interpretation of abstention. In section 4, we discuss some possibilities of interpretations and present specific cases that make existing rules compatible with our axioms. In section 5, we conclude and suggest further research on the use of this tool for referendum design.

2 Single binary-choice referenda: the model and actual rules

We consider the following scenario: a population of potential voters is called upon to vote in a single-issue referendum. We assume that the referendum question is phrased so that a vote for 'Yes' is a vote against the status quo, whereas a vote for 'No' supports the status quo⁴. Each individual has complete preferences over these two alternatives and can take one of three possible actions: vote 'Yes', vote 'No' or abstain (an implicit assumption is that there are no blank and null votes).

Formally, we assume an infinite population of potential agents indexed by the natural numbers (the set $\mathbb{N}$); let $\mathbb{N}$ be the class of non-empty finite subsets of $\mathbb{N}$; let $N \in \mathbb{N}$ denote the set of potential voters. Now let a set of actual voters be any $N' \subseteq N$ with $N' \in \mathbb{N}$. The set of abstainers is then $N \setminus N'$. Each agent $i \in N'$ is endowed with a rational⁵ preference relation R_i on the set of outcomes $A = \{Yes, No\}$. Let $\mathbb{R}$ denote the class of all such preference relations. We denote the preference profile of the set of actual voters by $R_{N'} = (R_i)_{i \in N'} \in \mathbb{R}^{N'}$. Let $R_{N \setminus N'}$ denote the preference profile of the set of abstainers where $R_{N \setminus N'} = (R_i)_{i \in N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$. We allow for the possibility of imposing a restriction on the preference domain

³The Condorcet principle defends the selection of the Condorcet winner, whenever that winner exists.

⁴We will derive the consequences of this assumption when we address referendum systems that are not symmetric with regard to the weights given to the status quo and the proposed change. In that case, and under our assumption, the system will not be neutral i.e. the names of the alternatives will be of relevance in the outcome.

⁵The implication of rationality in this setting is merely completeness, which does, however, allow for indifference.

of the set of abstainers, based on the remaining information. An electorate is then a set N of potential voters, where $N \in \mathbb{N}$, a set of actual voters N' , where $N' \in 2^N \setminus \emptyset$, an associated preference profile $R_{N'} = (R_i)_{i \in N'} \in \mathcal{R}^{N'}$, and a preference profile $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathcal{R}^{N \setminus N'}$ for the abstainers. We denote each electorate by $(N', N \setminus N', R_{N'}, R_{N \setminus N'})$. For simplicity, whenever $N' = N$, we simply denote the electorate by (N, R) . A voting rule is a mapping V that associates each electorate with a single outcome $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) \in A$.

Moulin (1988), in his study of the abstention paradox, also defines a voting rule as a function of the set of voters and their preferences, rather than their actions. A connection to actual systems would then entail the assumption that if a vote is actually cast, it reflects a preference i.e. if an agent votes, he votes for his most preferred alternative. In order to focus on the strategic problem of participation associated with the abstention paradox, another potential strategic issue - whether voters might have an incentive to vote for their worst alternative rather than their best - is ignored. With the purpose of connecting our preference-based voting rule with the action-based existing systems, we also introduce that behavioral assumption - a vote actually cast reveals a preference -, bearing in mind that the rules we study in this paper, as well as all rules that are actually used in any of the existing systems, are consistent with this assumption from a strategic viewpoint. For simplicity, we also assume that indifferent agents who vote choose either alternative randomly with equal probability.

We can now introduce some additional notation in order to express the basic rules used in most referenda and then proceed with the analysis.

Let $n = |N|$ and $n' = |N'|$. Given our behavioral assumptions, we define

$$\beta = \frac{|i \in N' : Yes P_i No| + \frac{1}{2} |i \in N' : Yes I_i No|}{|N'|}$$

to be the proportion of voters in N' that vote for Yes ⁶. In each of the current referendum systems, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = Yes$, defeating the status quo, if and only if some subset of the following conditions hold⁷:

1. The Majority (of voters) condition, $\beta > \lambda$. Yes wins only if the proportion of votes for Yes in the set of actual voters exceeds a constant λ .

⁶The inclusion of $\frac{1}{2} |i \in N' : Yes I_i No|$ in the numerator of β would be problematic if $|i \in N' : Yes I_i No|$ were odd. If that were the case, and the cardinality of the set were $2k + 1$, where k is an integer, we would simply have k agents voting for Yes and k agents voting for No . The $(2k + 1)$ th agent, that we call i , would vote so that $V(N, N', R_{N'}) = V(N, N' \setminus \{i\}, R_{N' \setminus \{i\}})$. Since the relevant assumption is that indifferent agents are not decisive, from now on we work as if n' were always even.

⁷We omit the case of referenda in federations, where a different additional condition applies: the majority of states/cantons.

2. The Voting Threshold condition, $\frac{n'}{n} > \tau$: *Yes* wins only if the proportion of actual voters in the population exceeds a constant τ .
3. The Majority Threshold condition, $\beta \frac{n'}{n} > \theta$: *Yes* wins only if the proportion of actual votes for *Yes* in the population exceeds a constant θ .

We can then use these conditions to define the main existing rules:

Definition 1 Rule $S(\lambda)$ is such that $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = \text{Yes}$ if and only if $\beta > \lambda$.

Definition 2 Rule $VT(\tau)$ is such that $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = \text{Yes}$ if and only if $\beta > \frac{1}{2}$ and $\frac{n'}{n} > \tau$.

Definition 3 Rule $MT(\theta)$ is such that $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = \text{Yes}$ if and only if $\beta > \frac{1}{2}$ and $\beta \frac{n'}{n} > \theta$.

The following table provides specific examples of these rules:

Type	Example	1. Majority	2. Voting Threshold	3. Majority Threshold
S	Ireland, Switzerland	$\beta > \frac{1}{2}$	-	-
S	Namibia	$\beta > \frac{2}{3}$	-	-
VT	Italy, Portugal	$\beta > \frac{1}{2}$	$\frac{n'}{n} > \frac{1}{2}$	-
MT	Germany (Land)	$\beta > \frac{1}{2}$	-	$\beta \frac{n'}{n} > \frac{1}{3}$
MT	Denmark (Act)	$\beta > \frac{1}{2}$	-	$\beta \frac{n'}{n} > \frac{3}{10}$
MT	Denmark (Const)	$\beta > \frac{1}{2}$	-	$\beta \frac{n'}{n} > \frac{2}{3}$
MT	Belarus	$\beta > \frac{1}{2}$	-	$\beta \frac{n'}{n} > \frac{2}{3}$

We should point out that we assume that a tie, that may occur if either of the conditions imposed by each system is met with equality, is broken lexicographically and *No* wins. This assumption has no effect on the main results and we therefore keep it throughout the analysis.

All systems require condition 1. The simple referendum, S , requires no additional condition. This is the case of Ireland where constitutional amendments have to be approved in a referendum, but where no minimal participation rate is set. In Switzerland, 50000 citizens may require a referendum on a normal act of parliament, and a majority of votes will suffice to approve it. With no threshold condition, there is perfect symmetry between *Yes* and *No*. The Namibian referendum for constitutional amendments, however, requires a majority of $\frac{2}{3}$ and symmetry is therefore lost.

Other systems require an additional turnout condition for the result to be binding. Since we defined *Yes* as a vote for a change and *No* as a vote for the

status quo, the failure to satisfy this condition implies that no change directly results from the referendum and therefore leads to the ultimate choice of *No* by the voting rule. The names of the alternatives become therefore relevant and, for *Yes* to effectively win, the turnout condition must be met - and the usual underlying rationale for this type of condition is based on the desire to ensure that the result is representative of the will of the population as a whole. The turnout condition can be formalized as either condition 2 or condition 3. A quick analysis of the table is enough to conclude that the *VT* referendum, that pairs Majority with Voting Threshold, implies the existence of an effective majority threshold and that the *MT* referendum, that pairs Majority with Majority Threshold, also implies the existence of a voting threshold. However, there is no equivalence between the two types of referenda. Looking at the Italian case, the conditions for the approval of *Yes* do imply the exact approval conditions in the German case. Yet, when Germany establishes that in each Land, the majority that supports a redesign of Lander boundaries has to represent over $\frac{1}{4}$ of the population, the only immediate consequence is that the voting threshold in that Land must also be at least $\frac{1}{4}$. The requirements of the *VT* referendum are therefore stronger (Figure 1).

Figure 1: All points in regions A, B and C satisfy the Majority condition; however, no point in C satisfies the Majority Threshold of $\frac{1}{4}$ and no point in either B or C satisfies the Voting Threshold of $\frac{1}{2}$.

The No-Show Paradox arises when an agent is better off abstaining than voting for his preferred choice. The *VT* referendum generates this paradox: if agent i is a *No* supporter and $\beta > \frac{1}{2}$, $\frac{n_i}{n} > \frac{1}{2}$ but $\frac{n_i-1}{n} \leq \frac{1}{2}$, it is optimal for i to abstain instead. The *S* and *MT* referenda do not cause this problem: in *S*, β will increase if a *No* supporter decides instead to abstain, and in *MT*, $\beta \frac{n_i}{n}$ remains unaltered by the additional abstention of a *No* supporter. The Italian and Portuguese referenda, by requiring participation, may actually discourage it. We formalize this notion of participation in our next section where we conduct an axiomatic study of this model.

3 Condorcet-Consistency, Representation and Abstention

The Condorcet principle defends the selection of the Condorcet winner - an alternative that defeats all others in a pairwise comparison - whenever that winner exists. Moulin (1988) shows that, although the Condorcet principle implies the No-Show Paradox among four or more candidates, this need not be the case for three candidates or less. Namely, for a single binary-choice referendum, a simple majority of voters together with some tie-breaking condition would be consistent with the Condorcet principle and would not cause the Paradox. However, Moulin makes no distinction between the set of actual voters and the whole population, in a situation where those two sets might not coincide. Our definition is explicitly based on all possible sets of actual voters.

Let $n_{YN}(N', R_{N'}) = |i \in N' : Yes P_i No| - |i \in N' : No P_i Yes|$ denote the balance of number of voters strictly preferring *Yes* to *No*⁸. *Yes* is a Condorcet winner if and only if $n_{YN}(N', R_{N'}) > 0$. *No* is a Condorcet winner if and only if $n_{YN}(N', R_{N'}) \leq 0$.

Definition 4 *V satisfies Condorcet Consistency if, for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$ and for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, $n_{YN}(N', R_{N'}) > 0 \Leftrightarrow V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = Yes$.*

This definition holds for all possible sets of actual voters where one of those sets is the whole population. However, rather than the rule satisfying Condorcet Consistency, it might instead be deemed desirable to apply that principle solely to the population as a whole: should the whole population vote, the majority would choose the outcome. This is, of course, a weaker requirement but will be useful once we introduce a notion of representation.

⁸Alternatively, we could have defined $n_{YN}(N, R) = |i \in N : Yes P_i No| + \frac{1}{2} |i \in N : Yes I_i No| - (|i \in N : No P_i Yes| + \frac{1}{2} |i \in N : Yes I_i No|)$ if $|i \in N : Yes I_i No|$ is even. Once more, if the cardinality of the set of indifferent agents were odd and equal to $2k+1$, we would include the additional $(2k+1)$ th agent so that the sign of n_{YN} remains unchanged.

Definition 5 *V satisfies Population Condorcet Consistency if, for all $N \in \mathbb{N}$, for all $R \in \mathbb{R}^N$, $n_{YN}(N, R_N) > 0 \Leftrightarrow V(N, R) = Yes$.*

Now, and so as to rule out the Abstention Paradox, we follow Moulin and introduce the Participation axiom, that requires that no agent strictly benefit from abstaining rather than voting for his most preferred alternative:

Definition 6 *V satisfies Participation if, for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$, for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, and for all $i \in N'$, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) R_i V(N' \setminus \{i\}, N \setminus (N' \setminus \{i\}), R_{N' \setminus \{i\}}, R_{N \setminus (N' \setminus \{i\})})$.*

Remark 1 *We have equivalence between $n_{YN}(N', R_{N'}) > 0$ and $\beta > \frac{1}{2}$. Therefore, the only rule that satisfies Condorcet-Consistency is $S(\frac{1}{2})$. Given that $S(\frac{1}{2})$ also satisfies Participation, our result is consistent with Moulin's (up to the tie-breaking assumption).*

At this point, we need to address additional issues that arise when the sets of potential and actual voters differ.

The first issue is the domain for preferences of abstainers. Given that these agents do not express their preferences through their votes, we may not wish to make any restrictions on those preferences, based either on the preferences of agents who do vote or on the information about the sets of potential and actual voters. In that case, we should apply the following axiom:

Definition 7 *If for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, and for all $R_{N'} \in \mathbb{R}^{N'}$, $D_{(N', N \setminus N', R_{N'})} = \mathbb{R}^{N \setminus N'}$, there is an unrestricted preference domain for the set of abstainers.*

Besides the domain for the preferences of abstainers, there is the additional matter of whether those preferences should have a direct effect on the voting outcome. Although we initially allowed the voting rule to select either 'Yes' or 'No' according to all the information that characterizes an electorate, it might be deemed desirable to exclude the preferences of abstainers from the information used by a voting rule. Indeed, no actual system explicitly depends on that piece of information - voting rules are defined as a function of the actions of the actual voters, the size of the population and the size of the set of actual voters. This is also the reason why, when we adapted our model to represent actual systems, we were able to bypass the debate on a positive theory of turnout⁹ and place no *a priori* restriction on the preferences of the abstaining segment of the population.

⁹Downs (1957) presented the seminal model of turnout, an issue more recently discussed, among many others, by Aldrich (1993) and Sen (1995).

With the purpose of ruling out the effect of 'irrelevant' potential voters, we should then introduce an additional invariance axiom: regardless of the preferences of individuals who do not vote, the voting rule should produce the same outcome.

Definition 8 *V satisfies Independence of Preferences of Abstainers if, for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$ and for all $R_{N \setminus N'}, R'_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = V(N', N \setminus N', R_{N'}, R'_{N \setminus N'})$.*

The next step in the analysis is to introduce an axiom that formalizes the concern with a representative outcome. Although this concern is usually expressed through the inclusion of a turnout condition in referenda, the underlying problem is not the number of agents who vote but rather whether the outcome coincides with the outcome that would result if the whole population were to reveal their preferences through voting.

One way to formalize this concern is to introduce a Reinforcement-type axiom, that we describe as Positive Representation - if the majority of actual voters prefers one alternative and the majority of the population also prefers that alternative, then the outcome selected by the voting rule for the set of actual voters should also be selected if the whole population were to vote.

Definition 9 *V satisfies Positive Representation if, for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$ and for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, whenever $\text{sign}(n_{Y_N}(N, R)) = \text{sign}(n_{Y_N}(N', R_{N'}))$, $V(N, R) = V(N', N \setminus N', R_{N'}, R_{N \setminus N'})$.*

Remark 2 $S(\frac{1}{2})$ is the unique rule that simultaneously satisfies Positive Representation, Condorcet Consistency and Participation, as well as Independence of Preferences of Abstainers.

Using Remark 1, we simply have to ensure that the rule satisfies Positive Representation, which follows from the application of Condorcet consistency to the population and the set of actual voters. Independence of Preferences of Abstainers is satisfied by the definition of S .

The Positive Representation axiom reflects the desire to ensure that when (the majorities of) the sets of potential and actual voters agree, the same choice is made by the voting rule. However, should there be disagreement between the two sets, the choice made for the set of actual voters might still be different from the choice made for the population. If we also want to avoid this possible discrepancy, we need to introduce another axiom, symmetric to the previous one:

Definition 10 V satisfies *Negative Representation* if, for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$ and for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, whenever $\text{sign}(n_{YN}(N, R)) \neq \text{sign}(n_{YN}(N', R_{N'}))$, $V(N, R) = V(N', N \setminus N', R_{N'}, R_{N \setminus N'})$.

However, with no restriction on the preferences of abstainers, this axiom is clearly incompatible with Condorcet Consistency:

Proposition 1 *With the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Negative Representation and Condorcet Consistency.*

Proof. Towards a contradiction, assume that there is a rule that simultaneously satisfies the axioms. Let $N \in \mathbb{N}$, $N' \in 2^N \setminus \emptyset$ and $R = (R_{N'}, R_{N \setminus N'}) \in \mathbb{R}^N$ be such that $n_{YN}(N', R_{N'}) > 0$ and $n_{YN}(N, R) \leq 0$. Then, by Negative Representation, $V(N, R) = V(N', N \setminus N', R_{N'}, R_{N \setminus N'})$, violating Condorcet Consistency. ■

Remark 3 *A corollary is that, with the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Negative Representation, Condorcet Consistency and Participation.*

We might want to check whether, with no restriction on the preference domain of the set of abstainers, the Negative Representation axiom is compatible with the weaker requirement of Population Condorcet Consistency. However, we find that if we also want the rule to be invariant with respect to preferences of abstainers, that compatibility would only hold if Participation were violated. The intuition for the result is that if we have a sufficiently small set of actual voters, the majority of the population might prefer either outcome; if we want the rule to be invariant with respect to the preferences of the non-voters and to simultaneously ensure Negative Representation, then we need to choose the alternative that the majority from the set of voters likes the *least*. But then, removing that majority from the set of actual voters yields a subset that is happy with the choice made for the initial set of voters, which in turn implies that the choice must change to the favorite outcome of the agents who left, contradicting Participation.

Proposition 2 *With the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Negative Representation, Population Condorcet Consistency, Independence of Preferences of Abstainers, and Participation.*

Proof. Towards a contradiction, assume that there is a rule that simultaneously satisfies the axioms. Consider the following electorate: $N \in \mathbb{N}$

and $n = 8$, $N' \in 2^N$ where $n' = 3$ and $N' = N'' \cup N' \setminus N''$ with $|N''| = 1$. $R_{N'}$ is such that $YesP_iNo$ for all $i \in N' \setminus N''$ and NoP_iYes for $i \in N''$. $R_{N \setminus N'}$ is such that NoP_iYes for all $i \in N \setminus N'$. Then, $n_{YN}(N', R_{N'}) = 1$ and $n_{YN}(N, R) = -4$. By Negative Representation and Population Condorcet Consistency, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = V(N, R) = No$. Now if each agent in $N' \setminus N''$ leaves N' , by a repeated application of Participation, $V(N'', N \setminus N'', R_{N''}, R_{N \setminus N''}) = No$. Now let $R'_{N \setminus N''}$ be such that $YesP_iNo$ for all $i \in N \setminus N''$. Again, by Negative Representation and Population Condorcet Consistency, $V(N'', N \setminus N'', R_{N''}, R'_{N \setminus N''}) = V(N, R) = Yes$, violating Independence of Preferences of Abstainers. ■

If we want the set of actual voters to accurately represent the population, then we need to ensure that the rule is such that, regardless of whether there is agreement between a majority of voters and a majority of the population, the outcome is the one that would occur were the whole population to vote. The underlying idea of the following axiom is therefore a notion of juxtaposition invariance:

Definition 11 *V satisfies Representation if it satisfies both Positive and Negative Representation: if for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathbb{R}^{N'}$ and for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subseteq \mathbb{R}^{N \setminus N'}$, $V(N, R) = V(N', N \setminus N', R_{N'}, R_{N \setminus N'})$.*

Remark 4 *A corollary of Proposition 1 is that with the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Representation and Condorcet Consistency - and this impossibility will naturally hold if Participation is also required.*

Our notion of representation combined with the unrestricted preference domain for the set of abstainers requires that the preferences of the agents who vote be such that their votes suffice to ascertain the will of the population as a whole; combining this with the Condorcet principle would therefore ensure that the procedure is democratic. This is then a preference-based notion of democracy - democracy means that the alternative that the whole population prefers is chosen -, distinct from the (potential-)action-based notion presented by Felsenthal and Machover (1997). However, this notion of democracy is too demanding and we reach the following negative result:

Theorem *With the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers.*

Proof. Towards a contradiction, assume that there is a rule that simultaneously satisfies the axioms. Let $N \in \mathfrak{N}$, $N' \in 2^N \setminus \emptyset$ with $n' < \frac{n}{2}$ and $n_{YN}(N', R_{N'}) > 0$. By Population Condorcet Consistency and Representation, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = \text{Yes}$ if and only if $n_{YN}(N, R) > 0$. Since $n' < \frac{n}{2}$, we can find $R'_{N \setminus N'}$ and $R''_{N \setminus N'}$ in $\mathfrak{R}^{N \setminus N'}$ such that $n_{YN}(N, (R_{N'}, R'_{N \setminus N'})) > 0$ and $n_{YN}(N, (R_{N'}, R''_{N \setminus N'})) \leq 0$. But then $V(N', N \setminus N', R_{N'}, R'_{N \setminus N'}) \neq V(N', N \setminus N', R_{N'}, R''_{N \setminus N'})$, violating Independence of Preferences of Abstainers. ■

The following result is a corollary of both the Theorem and Proposition 2:

Corollary *With the unrestricted preference domain for the set of abstainers, there is no rule that simultaneously satisfies Representation, Population Condorcet Consistency, Independence of Preferences of Abstainers, and Participation.*

Our theorem is a very intuitive result: we are looking for a voting rule that on the one hand cannot depend on the preferences of abstainers but on the other hand must reflect the wishes of the majority of the whole population, whatever the preferences of the segment who abstain may be. It is now clear that the systems described at the end of Section 2 may only satisfy Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers if appropriate restrictions on the preference domain of abstainers are introduced. Therefore, if the goal is to implement the will of the population as a whole and to ensure that the result of a referendum is the same for whatever preferences abstainers may have within their preference domain, that domain must be restricted¹⁰. Whether this restriction is based on any of the available information on the preferences of actual voters and on the sets of potential and actual voters, this means that preferences of abstainers that are not disclosed by the voting process are, or must be, nevertheless, interpreted. In our next section, we clarify this idea and show that most of the existing systems may indeed satisfy the three axioms, under specific interpretations of abstention.

4 Interpreting Abstention

We are now aware that when any referendum system, defined through a voting rule, intends to ensure an accurate representation of the preferences of the population, assumptions on the preferences of abstainers are inherently

¹⁰In this sense, our result bears a resemblance to Arrow's (1963) seminal result.

made. We propose instead that the process be reversed: that those assumptions be consciously decided upon and then used as a means to define the referendum rule.

In this section, we therefore invert the usual process and show that for appropriate restrictions on the preferences of the abstaining agents, most of the referendum rules that are actually applied may satisfy Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers - and actually, that a restriction on the *proportion* of abstainers that favor *Yes* will suffice. This type of restriction amounts to a specific interpretation of abstention, but although we redirect the problem of defining a referendum system to the problem of interpreting abstention, we do not intend to provide any recommendation about how to solve the latter.

One possible view on abstention is that agents who abstain are indifferent towards the issue, given that they had the same prior power (to vote) and decided not to exercise it; in that case, those agents are dummies and can be ignored. Another possible interpretation is specific to referenda: since referenda are held to decide on a change to the status quo, people who abstain do so because they are comfortable with the status quo and abstention is in that case interpreted as support for *No*¹¹. A legal analogy might be useful in understanding these cases: whereas in the latter, the burden of proof is on *Yes*, in the former, both *Yes* and *No* are innocent until proven guilty. Each of these views depends however on a specific positive theory of turnout and on specific assumptions about independence or homogeneity of the preferences of the population. Like Braham and Steffen (2001), we leave possible positive (and normative) statements about this issue to future research. Instead, we merely consider the main types of referenda and offer possible interpretations that could justify each of them as representative of the wishes of the population.

We denote the proportion of abstainers in $N \setminus N'$ that favor *Yes* by

$$\alpha = \frac{|i \in N \setminus N' : Yes P_i No| + \frac{1}{2} |i \in N \setminus N' : Yes I_i No|}{|N'|}.$$

Since N and N' are finite sets with integer cardinalities, we can only have $\alpha \in \{0, \frac{1}{n-n'}, \frac{2}{n-n'}, \dots, 1\}$. In order to simplify the exposition, however, we use $\alpha \in [0, 1]$. Still, for each α we consider in this section, there is an appropriate α' that may be written as $\frac{k}{n-n'}$ for some $k = 0, 1, \dots, n - n'$ such that $|\alpha' - \alpha| \leq \frac{1}{2(n-n')}$, yielding the same conclusions.

There are three main axioms we are interested in. All the rules that we study satisfy Independence of Preferences of Abstainers. We can now apply

¹¹...or a vote against holding the referendum. This must, however, be specific to the referendum scenario because otherwise, in a general election, abstention would also have to be interpreted as votes for the incumbent party - or, alternatively, against democracy itself.

Figure 2: For the $S(\frac{1}{2})$ referendum, α must be in the shaded region for all possible economies. Two functions that would satisfy the requirement are $\alpha = \frac{1}{2}$ (compatible with the idea of indifference of abstainers with respect to the outcome) and $\alpha = \beta$ (compatible with the idea that the voters and the abstainers share the same preferences).

this additional notation in order to condense the other two requirements of Representation and Population Condorcet Consistency.

Remark 5 *V satisfies Representation and Population Condorcet Consistency if for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathcal{R}^{N'}$ and for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})} \subset \mathcal{R}^{N \setminus N'}$, $V(N', N \setminus N', R_{N'}, R_{N \setminus N'}) = \text{Yes}$ if and only if $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n') \Leftrightarrow \alpha > (\frac{n}{2} - \beta n') / (n - n')$.*

We begin by analyzing $S(\lambda)$. It is clear that for the Namibian case or for any $S(\lambda)$ such that $\lambda \neq \frac{1}{2}$, these axioms are not compatible since Population Condorcet Consistency alone will fail if the whole population votes and β is a value between λ and $\frac{1}{2}$. However, for $\lambda = \frac{1}{2}$, there are several possible values α may take such that S satisfies the three axioms. In fact, there are several possible functions $\alpha(\beta)$ such that for any combination of n and n' , this result holds (Fig. 2).

Claim 1.1 If for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathcal{R}^{N'}$, $D_{(N', N \setminus N', R_{N'})}$ is such that for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})}$ we have $\alpha = \frac{1}{2}$,

then $S(\frac{1}{2})$ satisfies Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers. Furthermore, $S(\frac{1}{2})$ satisfies Condorcet Consistency.

Proof. Using Remark 5, it is straightforward that if $\alpha = \frac{1}{2}$, then $\beta > \frac{1}{2}$ is equivalent to $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$. ■

As we mentioned above, one possible interpretation of abstention is that all agents who abstain are indifferent towards the issue¹². This is the case considered in Claim 1.1, where the preferences of the population are then restricted to be an additive extension of the preferences of the actual voters. Another possibility is that the population is restricted to be a multiplicative extension of the set of voters and that the preferences of the set of voters parallel the preferences of the set of abstainers. This is the case considered in the following Claim:

Claim 1.2 If for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathfrak{R}^{N'}$, $D_{(N', N \setminus N', R_{N'})}$ is such that for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})}$ we have $\alpha = \beta$, then $S(\frac{1}{2})$ satisfies Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers. Furthermore, $S(\frac{1}{2})$ satisfies Condorcet Consistency.

Proof. Using Remark 5, it is straightforward that if $\alpha = \beta$, then $\beta > \frac{1}{2}$ is equivalent to $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$. ■

An analysis of Figure 2 allows us to conclude that in fact any combination of the additive and multiplicative extensions would result in a restriction that makes $S(\frac{1}{2})$ compatible with the three axioms.

Neither of these extensions would, however, represent appropriate restrictions for the $VT(\tau)$ system. Instead, the additional voting threshold condition allows for the following restriction:

Claim 2 If for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathfrak{R}^{N'}$, $D_{(N', N \setminus N', R_{N'})}$ is such that for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})}$ we have

$$\alpha = \begin{cases} \frac{1}{2} & \text{if } \beta \leq \frac{1}{2} \\ (1 - 2\tau\beta) / [2(1 - \tau)] & \text{if } \beta > \frac{1}{2} \end{cases},$$

then $VT(\tau)$ with $\tau \leq \frac{1}{2}$ satisfies Representation and Population Condorcet Consistency.

¹²Given our behavioral assumptions, this would also be the relevant case, should the concern be a notion of average (rather than exact) representation where equal probabilities are assigned to each (unrestricted) preference profile of abstainers.

Proof. Imposing $\tau \leq \frac{1}{2}$ ensures that $\alpha \in [0, 1]$. Using Remark 5, we simply need to show that if $\alpha = \frac{1}{2}$ for $\beta \leq \frac{1}{2}$ and $\alpha = (1 - 2\tau\beta) / [2(1 - \tau)]$ for $\beta > \frac{1}{2}$, then $\beta > \frac{1}{2}$ and $\frac{n'}{n} > \tau$ are equivalent to $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$. If $\beta \leq \frac{1}{2}$, $\alpha = \frac{1}{2}$ and $\beta n' + \alpha(n - n') \leq (1 - \beta)n' + (1 - \alpha)(n - n')$. If $\beta > \frac{1}{2}$, $\alpha = (1 - 2\tau\beta) / [2(1 - \tau)]$ and $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$ if and only if $\frac{n'}{n} > \tau$. ■

For the Italian and Portuguese referenda (or for decision-making in committees with a quorum requirement of 50%), depicted in Figures 3 and 4, we then have that $\alpha = \frac{1}{2}$ if $\beta \leq \frac{1}{2}$, and $\alpha = 1 - \beta$ if $\beta > \frac{1}{2}$, is an appropriate restriction (in fact, for $\beta > \frac{1}{2}$, $\alpha = 1 - \beta$ is the only function of β that would satisfy the conditions for all n, n'). This type of referendum therefore imposes an asymmetry between *Yes* and *No*, attributing a greater weight to the status quo. Whether it is a reasonable choice to have the proportion of abstainers favorable to *Yes* depend negatively on the proportion of voters with the same preferences is beyond the scope of our analysis. In terms of the participation requirement, however, we have already seen that this system is problematic¹³.

The $MT(\theta)$ system also fails to allow for the additive and multiplicative extensions that could be applied to $S(\frac{1}{2})$. Instead, the additional majority threshold condition allows for the following restriction:

Claim 3 If for all $N \in \mathbb{N}$, for all $N' \in 2^N \setminus \emptyset$, for all $R_{N'} \in \mathcal{R}^{N'}$, $D_{(N', N \setminus N', R_{N'})}$ is such that for all $R_{N \setminus N'} \in D_{(N', N \setminus N', R_{N'})}$ we have

$$\alpha = \begin{cases} \frac{1}{2} & \text{if } \beta \leq \frac{1}{2} \\ \min \left\{ \left[(1 - 2\theta)n \right] / [2(n - n')], \frac{1}{2} \right\} & \text{if } \beta > \frac{1}{2} \end{cases},$$

then $MT(\theta)$ with $\theta \leq \frac{1}{2}$ satisfies Representation, Population Condorcet Consistency and Independence of Preferences of Abstainers.

Proof. Once more, imposing $\theta \leq \frac{1}{2}$ ensures that $\alpha \in [0, 1]$.

Using Remark 5, we simply need to show that if $\alpha = \frac{1}{2}$ for $\beta \leq \frac{1}{2}$ and $\alpha = \min \left\{ \left[(1 - 2\theta)n \right] / [2(n - n')], \frac{1}{2} \right\}$ for $\beta > \frac{1}{2}$, then $\beta > \frac{1}{2}$ and $\beta \frac{n'}{n} > \theta$ are equivalent to $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$. If $\beta \leq \frac{1}{2}$, $\alpha = \frac{1}{2}$ and $\beta n' + \alpha(n - n') \leq (1 - \beta)n' + (1 - \alpha)(n - n')$. If $\beta > \frac{1}{2}$ and $\left[(1 - 2\theta)n \right] / [2(n - n')] \geq \frac{1}{2} \Leftrightarrow \frac{n'}{2n} \geq \theta$, then $\alpha = \frac{1}{2}$ and $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$. If $\beta > \frac{1}{2}$ and $\left[(1 - 2\theta)n \right] / [2(n - n')] < \frac{1}{2} \Leftrightarrow \frac{n'}{2n} < \theta$, then $\alpha = \left[(1 - 2\theta)n \right] / [2(n - n')]$ and $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$ if and only if $\beta \frac{n'}{n} > \theta$. The result then follows because $\beta > \frac{1}{2}$ and $\frac{n'}{2n} \geq \theta$ imply $\beta \frac{n'}{n} > \theta$. ■

¹³It should be noted, in any case, that there might be an endogeneity factor at work here: if participation of *No* supporters is discouraged, there might be a reason to believe that the preferences of abstainers are not homogeneous.

Figure 3: This is a depiction of $VT(1/2)$, when $n' \leq n/2$. In this case, $(n/2 - \beta n') / (n - n') \geq 1 - \beta$ for $\beta > 1/2$ and the rule selects *No* for all β .

Figure 4: Here, we have $n' > n/2$. In this case, $(n/2 - \beta n') / (n - n') < 1 - \beta$ and the rule selects *Yes* for $\beta > 1/2$. Having $\alpha = 1/2$ if $\beta \leq 1/2$ and $\alpha = 1 - \beta$ if $\beta > 1/2$ accurately represents the decisions in both cases.

Figure 5: This is the case for $MT(1/2)$ where $\frac{n}{4(n-n')} \leq \frac{1}{2}$ or, equivalently, $n' \leq \frac{n}{2}$. *Yes* wins only if β is above $\frac{n}{4n'}$, the value that corresponds to the intersection of $(\frac{n}{2} - \beta n') / (n - n')$ and $\frac{n}{4(n-n')}$. For the case where $n' > \frac{n}{2}$, we can simply use $\alpha = \frac{1}{2}$ and go back to Figure 2.

For $MT(1/2)$, the case applied to each German Land in the Lander redesign question (Figure 5), we can then have that $\alpha = 1/2$ for $\beta \leq 1/2$, and $\alpha = \min \left\{ \frac{n}{4(n-n')}, \frac{1}{2} \right\}$ for $\beta > 1/2$, is an appropriate restriction. In fact, for $\beta > 1/2$, and $\frac{n}{4(n-n')} \leq \frac{1}{2}$, $\alpha = \frac{n}{4(n-n')}$ is the only restriction that can be expressed as function of n and n' that is constant with respect to β . In this case, the proportion of abstainers that favor *Yes* is then increasing in n' for the same n and decreasing in n for the same n' (and decreasing with respect to the number of abstainers). Again, an asymmetry is introduced favoring the status quo; this asymmetry, however, introduces no conflict with Participation¹⁴.

¹⁴Although we only presented one possible restriction on α that would make each system compatible with our axioms, since all such restrictions are equivalent in terms of decision, we can check whether each of these systems satisfies Participation by simply analyzing the one restriction we used.

Denoting by $\alpha(\beta, n, n')$ the restriction on the preferences of the set of abstainers, we encompass all of the aforementioned cases. Participation is then satisfied if for all (β, n, n') , $\alpha = \alpha(\beta, n, n')$ and $\alpha' = \alpha(\beta', n, n' + 1)$ are such that neither of the following hold:

1. With $\beta' = \beta n' / (n' + 1)$, both $\beta n' + \alpha(n - n') \leq (1 - \beta)n' + (1 - \alpha)(n - n')$ and $\beta' n' +$

All of the main referendum systems may thus be interpreted as attempts to effectively represent the will of the whole population. However, given that only a fraction of that population votes, the outcome of the referendum will only be representative if the preferences of the non-voters are restricted. Awareness of this implication seems of relevance for referendum design: rather than starting by deciding whether or not to impose *ad hoc* turnout conditions, where either decision necessarily entails specific interpretations of abstention, a conscious decision on an adequate interpretation of abstention should be made instead, leading to the definition of the voting rule. We therefore offer the interpretation of abstention as a tool for the design of a representative referendum.

5 Conclusion

We analyze single binary-choice voting rules and identify the presence of the No-Show paradox in this simple setting, as a consequence of specific turnout and quorum conditions that are included, respectively, in actual referendum systems and in rules for decision-making in committees. Given that these conditions are meant to ensure a representative outcome, we try to formalize this concern with representation in the presence of abstention and reach our main impossibility result: no voting rule can ensure an accurate representation of the wishes of the population if no restrictive assumptions are made on the preferences of abstainers. A direct consequence of this result is that representation may be achieved for appropriate restrictions on the preference domain of the set of abstainers. We then proceed with the analysis of the main referendum systems and show that such restrictions do exist for most of those systems.

The main purpose of our paper is, however, to provide a tool for referendum design. The definition of a voting system that intends to be representative inevitably results in restrictions on the preferences of non-voters. Unawareness of this implication may lead to the imposition of restrictions that do not satisfy any type of criteria but cause such problems as the No-

$$\alpha'(n - n' - 1) > (1 - \beta')n' + (1 - \alpha')(n - n' - 1);$$

2. With $\beta' = \beta(n' + 1)/n'$, both $\beta n' + \alpha(n - n') > (1 - \beta)n' + (1 - \alpha)(n - n')$ and $\beta'(n' + 1) + \alpha'(n - n' - 1) \leq (1 - \beta')(n' + 1) + (1 - \alpha')(n - n' - 1)$.

The first condition represents the case where a *No* supporter decides to vote, changing the result to *Yes* and the second condition represents the symmetric case. It is straightforward to check, using the appropriate $\alpha(\beta, n, n')$ that these conditions are never possible for both $S(\frac{1}{2})$ and $MT(\theta)$ with $\theta \leq \frac{1}{2}$ but that although $VT(r)$ with $r \leq \frac{1}{2}$ never satisfies the second condition, there are indeed cases where the first one holds: just let $\beta > \frac{n'+1}{2n'}$ and let $2n' \leq n < 2(n' + 1)$.

Show paradox: abstention may be encouraged by how a system chooses to interpret that abstention. We therefore suggest making a conscious choice on how abstention is to be interpreted and using that choice as a tool to derive the corresponding referendum rule.

We do not, however, provide criteria for these choices. They may be based on a positive theory of turnout that may in turn depend on prior assumptions on the preferences of the potential voters, as well as on the distribution of pure costs and benefits of voting among the population; we might also have a decision-maker performing a Bayesian update of the proportion of abstainers who support the status quo based on the information provided by the sample of voters, and on some prior that could reflect a positive degree of risk aversion and be more favorable to the status quo; also, given that the main results of the paper apply to all binary-choice scenarios, it would be interesting to analyze how different explanations of turnout that may apply to small and large numbers would suggest different systems for referenda and for decision-making in committees (where personal interaction is possible and strategic coalition behaviour may occur). All these questions, as well as possible justifications for the choices implied by the main voting systems, are open to further research.

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